

CP Violation in $B \rightarrow \rho\pi$: New Physics Signals

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Abstract

A Dalitz-plot analysis of $B_d^0(t) \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$ decays allows one to obtain the CP-violating phase α . In addition, one can extract the various tree (T) and penguin (P) amplitudes contributing to these decays. By comparing the measured value of $|P/T|$ with the theoretical prediction, one can detect the presence of physics beyond the standard model.

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A great many methods have been proposed for obtaining information about the CP phases α , β and γ of the unitarity triangle [1]. Almost all of these involve CP-violating asymmetries in hadronic B decays [2]. The aim is to test the standard model (SM) explanation of CP violation, and hopefully find evidence for physics beyond the SM.

The cleanest methods (i.e. those in which the theoretical hadronic uncertainties are very small) involve B decays which are dominated by a single amplitude, such as $B_d^0(t) \rightarrow J/\psi K_S$. However, many decays receive contributions from both tree and penguin diagrams with different weak phases [3]. A-priori, one would think that one cannot obtain clean phase information from such decays. Fortunately, techniques have been developed for removing the unwanted “penguin pollution.” For example, an isospin analysis of $B \rightarrow \pi\pi$ decays allows one to remove this contamination and obtain $\sin 2\alpha$ cleanly [4], albeit with discrete ambiguities.

In fact, this isospin analysis gives us even more information. In particular, one can also obtain the magnitudes and relative phases of the tree (T) and penguin (P) amplitudes in $B_d^0 \rightarrow \pi^+\pi^-$ [5]. It is therefore possible to compare the experimental value of $|P/T|$ with that predicted by theory. If a significant discrepancy is observed, it would signal new physics [6].

An alternative technique for obtaining α involves $B \rightarrow \rho\pi$ decays. By performing a Dalitz-plot analysis of $B_d^0(t) \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$ decays, one can remove the penguin contributions from $B \rightarrow \rho\pi$ decays and obtain α [7]. Compared to $B \rightarrow \pi\pi$, the advantage of this method is that it is possible to extract both $\sin 2\alpha$ and $\cos 2\alpha$, so that one obtains 2α with no discrete ambiguity. Another advantage is that it is not necessary to measure processes involving two final-state π^0 mesons. The disadvantage of this method is that one must understand the continuum background to such decays with considerable accuracy, as well as the correct description of $\rho \rightarrow \pi\pi$ decays, and these may be difficult.

Here too there is enough information to obtain the magnitudes and relative phases of the tree and penguin amplitudes. Thus, one can measure $|P/T|$ in $B \rightarrow \rho\pi$. As in $B \rightarrow \pi\pi$, a comparison of this ratio with the theoretical prediction can reveal the presence of new physics. In this paper we perform such an analysis. As we will show, the $B \rightarrow \rho\pi$ method has two advantages compared to $B \rightarrow \pi\pi$ for searching for physics beyond the SM in this way. First, the fact that there is no discrete ambiguity in 2α improves the prospects for finding new physics. Second, the $|P/T|$ ratio is expected to be smaller than in $B \rightarrow \pi\pi$, which makes it easier to see a new-physics signal, should it be present.

We begin with a brief review of the $B \rightarrow \rho\pi$ Dalitz-plot analysis within the SM [7]. There are five $B \rightarrow \rho\pi$ amplitudes which satisfy a pentagon isospin relation. All amplitudes receive contributions from both tree and $\bar{b} \rightarrow \bar{d}$ penguin amplitudes. The tree amplitude is proportional to $V_{ub}^*V_{ud}$, while the penguin amplitude has contributions from internal u , c and t quarks, proportional to $V_{ub}^*V_{ud}$, $V_{cb}^*V_{cd}$ and $V_{tb}^*V_{td}$, respectively. Using the unitarity of the CKM matrix, $V_{td}^*V_{tb} + V_{cd}^*V_{cb} + V_{ud}^*V_{ub} = 0$,

we can eliminate the c -quark contribution. Furthermore, the piece proportional to $V_{ub}^* V_{ud}$ can be absorbed into the tree amplitude. Thus, the penguin amplitude includes only the piece proportional to $V_{td}^* V_{tb}$. It is convenient to rescale the amplitudes by $e^{i\beta}$, leading to the following expressions for the amplitudes:

$$\begin{aligned}
S_{+0} &\equiv e^{i\beta} \sqrt{2} A(B^+ \rightarrow \rho^+ \pi^0) = T^{+0} e^{-i\alpha} + P^{+0} , \\
S_{0+} &\equiv e^{i\beta} \sqrt{2} A(B^+ \rightarrow \rho^0 \pi^+) = T^{0+} e^{-i\alpha} + P^{0+} , \\
S_{+-} &\equiv e^{i\beta} A(B^0 \rightarrow \rho^+ \pi^-) = T^{+-} e^{-i\alpha} + P^{+-} , \\
S_{-+} &\equiv e^{i\beta} A(B^0 \rightarrow \rho^- \pi^+) = T^{-+} e^{-i\alpha} + P^{-+} , \\
S_{00} &\equiv e^{i\beta} 2A(B^0 \rightarrow \rho^0 \pi^0) = S_{+0} + S_{0+} - S_{+-} - S_{-+} ,
\end{aligned} \tag{1}$$

where $P^{0+} = -P^{+0}$. In the above we have explicitly written the weak phase α , while the T_i and the P_i include strong phases. (Throughout the paper, we use the subscript ‘ i ’ to denote all of the $\rho\pi$ charge combinations: $i = +0, 0+, +-, -+, 00$.) The corresponding amplitudes for the CP-conjugate processes, \bar{S}_i , are obtained by changing the signs of the weak phases.

The key point is that all of the neutral $B_d^0 \rightarrow \rho\pi$ amplitudes contribute to $B_d^0 \rightarrow \pi^+ \pi^- \pi^0$. We can therefore write

$$A(B_d^0 \rightarrow \pi^+ \pi^- \pi^0) = f^+ S_{+-} + f^- S_{-+} + f^0 S_{00}/2 , \tag{2}$$

where the f^i are the kinematic distribution functions for the pions produced in the decay of the ρ^i . The \bar{B}_d^0 mesons can decay to the same final state:

$$A(\bar{B}_d^0 \rightarrow \pi^+ \pi^- \pi^0) = f^- \bar{S}_{+-} + f^+ \bar{S}_{-+} + f^0 \bar{S}_{00}/2 . \tag{3}$$

The time-dependent measurement of the Dalitz plot for $B_d^0(t) \rightarrow \pi^+ \pi^- \pi^0$ then allows one to extract the magnitudes and relative phases of each of the f^i , S_{ij} and \bar{S}_{ij} in Eqs. (2) and (3) [7]. By taking the ratio of the relations

$$\begin{aligned}
S_{+-} + S_{-+} + S_{00} &= (T^{+0} + T^{0+}) e^{-i\alpha} , \\
\bar{S}_{+-} + \bar{S}_{-+} + \bar{S}_{00} &= (T^{+0} + T^{0+}) e^{i\alpha} ,
\end{aligned} \tag{4}$$

one obtains $e^{-2i\alpha}$. We therefore see that, using this method, the CP phase 2α can be extracted with no ambiguity.

It is also possible to obtain the tree and penguin contributions to the amplitudes in Eq. (1). We define the following observables:

$$\begin{aligned}
B_i &\equiv \frac{1}{2}(|A_i|^2 + |\bar{A}_i|^2) , \\
a_i &\equiv \frac{|S_i|^2 - |\bar{S}_i|^2}{|S_i|^2 + |\bar{S}_i|^2} , \\
2\alpha_{eff}^i &\equiv \text{Arg}(\bar{S}_i S_i^*) .
\end{aligned} \tag{5}$$

Here B_i , a_i and $2\alpha_{eff}^i$ are, respectively, the branching ratio, direct CP asymmetry, and measure of indirect CP violation for each decay. We remark that each of $2\alpha_{eff}^{+0}$ and $2\alpha_{eff}^{0+}$ are automatically zero since they involve charged B decays. (Note: the indirect CP asymmetry is usually written with an explicit mixing phase $q/p = e^{-2i\beta}$. This phase is removed when one rescales the amplitudes by $e^{i\beta}$ as in Eq. (1).) We have

$$\begin{aligned} S_i - \bar{S}_i &= -2i \sin \alpha T_i , \\ S_i e^{i\alpha} - \bar{S}_i e^{-i\alpha} &= -2i \sin \alpha P_i . \end{aligned} \quad (6)$$

It is then straightforward to obtain $|T_i|^2$ and $|P_i|^2$:

$$\begin{aligned} |T_i|^2 &= R_i \frac{1 - \sqrt{1 - a_i^2} \cos 2\alpha_{eff}^i}{1 - \cos 2\alpha} , \\ |P_i|^2 &= R_i \frac{1 - \sqrt{1 - a_i^2} \cos(2\alpha_{eff}^i - 2\alpha)}{1 - \cos 2\alpha} . \end{aligned} \quad (7)$$

where

$$R_i \equiv \frac{(|S_i|^2 + |\bar{S}_i|^2)}{2} . \quad (8)$$

Note that R_i is proportional to B_i [Eq. (5)]. The proportionality constant depends on which decay is being considered, see Eq. (1).

Suppose now that there is physics beyond the SM. If present, it will affect mainly $B_d^0 - \bar{B}_d^0$ mixing and/or the $\bar{b} \rightarrow \bar{d}$ penguin amplitude. In the SM, the weak phase of $B_d^0 - \bar{B}_d^0$ mixing (β) is equal to that of the t -quark contribution to the $\bar{b} \rightarrow \bar{d}$ penguin. This is reflected in the fact that the weak phase multiplying the term P_i in Eq. (1) is zero. If new physics is present, these two weak phases may be different. One can take this possibility into account by including a new-physics phase θ_{NP} in the $B \rightarrow \rho\pi$ amplitudes:

$$\begin{aligned} S_{+0} &= T^{+0} e^{-i\alpha} + P^{+0} e^{-i\theta_{NP}} , \\ S_{0+} &= T^{0+} e^{-i\alpha} + P^{0+} e^{-i\theta_{NP}} , \\ S_{+-} &= T^{+-} e^{-i\alpha} + P^{+-} e^{-i\theta_{NP}} , \\ S_{-+} &= T^{-+} e^{-i\alpha} + P^{-+} e^{-i\theta_{NP}} , \\ S_{00} &= S_{+0} + S_{0+} - S_{+-} - S_{-+} . \end{aligned} \quad (9)$$

The extraction of α is unchanged by the presence of the new-physics parameter θ_{NP} (though its value may include new contributions to $B_d^0 - \bar{B}_d^0$ mixing). However, the expressions for T_i and P_i are modified. We now have

$$\begin{aligned} S_i e^{i\theta_{NP}} - \bar{S}_i e^{-i\theta_{NP}} &= -2i \sin(\alpha - \theta_{NP}) T_i , \\ S_i e^{i\alpha} - \bar{S}_i e^{-i\alpha} &= -2i \sin(\alpha - \theta_{NP}) P_i , \end{aligned} \quad (10)$$

so that

$$\begin{aligned}
|T_i|^2 &= R_i \frac{1 - \sqrt{1 - a_i^2} \cos(2\alpha_{eff}^i - 2\theta_{NP})}{1 - \cos(2\alpha - 2\theta_{NP})} , \\
|P_i|^2 &= R_i \frac{1 - \sqrt{1 - a_i^2} \cos(2\alpha_{eff}^i - 2\alpha)}{1 - \cos(2\alpha - 2\theta_{NP})} .
\end{aligned} \tag{11}$$

The expressions for the T_i and P_i are therefore altered in the presence of new physics. Thus, by comparing the measured value of a particular $|P/T|$ with that predicted by theory (within the SM), we can detect the presence of a nonzero θ_{NP} . (It is also possible for new physics to affect the magnitudes of the T_i and P_i . This possibility is implicitly included in our method.)

The first step is therefore to evaluate the theoretical value of $|P/T|$. However, there are many $|P/T|$ ratios that can be considered. We concentrate only on the (color-allowed) neutral decays $B_d^0 \rightarrow \rho^\pm \pi^\mp$. There are several reasons for this. First, the Dalitz plots for the charged B decays are much more difficult to obtain since they require the detection of two π^0 's. Second, the branching ratio for the color-suppressed decay $B_d^0 \rightarrow \rho^0 \pi^0$ is expected to be quite a bit smaller than those of $B_d^0 \rightarrow \rho^\pm \pi^\mp$. Finally, below we will use QCD factorization to estimate the theoretical size of the $|P/T|$ ratios, and nonfactorizable effects are expected to be small for color-allowed decays.

The value of $|P/T|$ for $B_d^0 \rightarrow \rho^\pm \pi^\mp$ has been calculated in the literature. In Ref. [8], this was done using naive factorization and including only the t -quark contribution to the $\bar{b} \rightarrow \bar{d}$ penguin. A more recent computation has been done by Beneke and Neubert [9] in the context of QCD factorization [10]. Since QCD factorization is a state-of-the-art framework, using expansions in $1/m_b$ and α_s , we will follow this approach. (It should be noted, however, that the $|P/T|$ ranges given in Refs. [8] and [9] are quite similar.)

We define

$$r^{+-} \equiv \left| \frac{P^{+-}}{T^{+-}} \right| , \quad r^{-+} \equiv \left| \frac{P^{-+}}{T^{-+}} \right| . \tag{12}$$

Ref. [9] gives

$$r^{+-} = 0.10_{-0.04}^{+0.06} , \quad r^{-+} = 0.10_{-0.05}^{+0.09} . \tag{13}$$

The errors come principally from three sources: the values of $|V_{ub}|$ and m_s , and the size of weak annihilation effects. Note that the two ratios are determined by very different dynamics, so that their near equality is a numerical accident.

It is now necessary to decide on the numerical ranges to use for r^{+-} and r^{-+} in the analysis. Since the goal is to search for physics beyond the SM, it is important to be as conservative as possible. With this in mind, we will take the theoretical ranges for r^{+-} and r^{-+} within the SM to be

$$0.05 < r^{+-} < 0.25 , \quad 0.05 < r^{-+} < 0.25 . \tag{14}$$

The above ranges are larger than those given in Eq. (13), particularly on the upper side. We note that QCD factorization cannot account for the observed $B \rightarrow \pi\pi$ and $B \rightarrow \rho\pi$ branching ratios [9]. Assuming no new physics — and the analysis of this paper can be used to test for such effects — there must be some contribution which is larger than its QCD factorization value. The enlarged ranges of Eq. (14) take this into account, as well as potential underestimates of factorizable errors (e.g. electroweak-penguin effects) and nonfactorizable effects. With the ranges of Eq. (14), a significant discrepancy between the measured value of $|P/T|$ and its SM prediction will clearly be a sign of new physics. That is, within the SM, we expect

$$0.05 < \sqrt{\frac{1 - \sqrt{1 - a_i^2} \cos(2\alpha_{eff}^i - 2\alpha)}{1 - \sqrt{1 - a_i^2} \cos 2\alpha_{eff}^i}} < 0.25 , \quad (15)$$

for $i = +-, -+$. If it is found that the observables do not respect this inequality, this points to the presence of physics beyond the SM.

We note in passing that, while the $|P/T|$ range in $B \rightarrow \rho\pi$ is $\sim 10\%$, in $B \rightarrow \pi\pi$ it is expected to be $\sim 20\text{--}30\%$ [6, 9, 11]. We therefore conclude that the penguin pollution is likely to be more significant in $B \rightarrow \pi\pi$ [8]. Thus, if new physics is present, it will be easier to detect in $B \rightarrow \rho\pi$.

As noted earlier, the CP phase 2α can be extracted from the $B \rightarrow \rho\pi$ method. However, this is not easy experimentally. In our analysis we therefore consider the possibility that only $\sin 2\alpha$ is measured (in which case one obtains 2α with a twofold ambiguity), as well as the case where 2α is known without ambiguity. In both scenarios, we consider two possible ranges for 2α : (i) $120^\circ \leq 2\alpha \leq 135^\circ$, (ii) $165^\circ \leq 2\alpha \leq 180^\circ$, which can be considered to take into account the experimental errors in the measurements.

Our results are shown in Fig. 1. We consider the two ranges for 2α given above. For each of these ranges, Fig. 1 shows the (correlated) allowed values of $2\alpha_{eff}^i$ and a_i ($i = +-, -+$) that are consistent, within the SM, with the assumed range for 2α and the theoretical range for $|P^i/T^i|$ [Eq. (14)]. If only $\sin 2\alpha$ has been measured, then, for a given range of 2α , both regions in Fig. 1 are allowed. If $\cos 2\alpha$ can also be measured, then the left-hand region can be removed. In either of these scenarios, if the measured values of the observables do not lie within the SM region, this means that new physics — i.e. a nonzero θ_{NP} — is present. As can be seen from Fig. 1, the new-physics region is quite large, so that we have a good chance of detecting the new physics via this method, should it be present.

If no signal for new physics is detected, one can place an upper limit on the value of θ_{NP} via

$$0.05 < \sqrt{\frac{1 - \sqrt{1 - a_i^2} \cos(2\alpha_{eff}^i - 2\alpha)}{1 - \sqrt{1 - a_i^2} \cos(2\alpha_{eff}^i - 2\theta_{NP})}} < 0.25 . \quad (16)$$

To summarize, the measurement of the Dalitz plot of $B_d^0(t) \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$

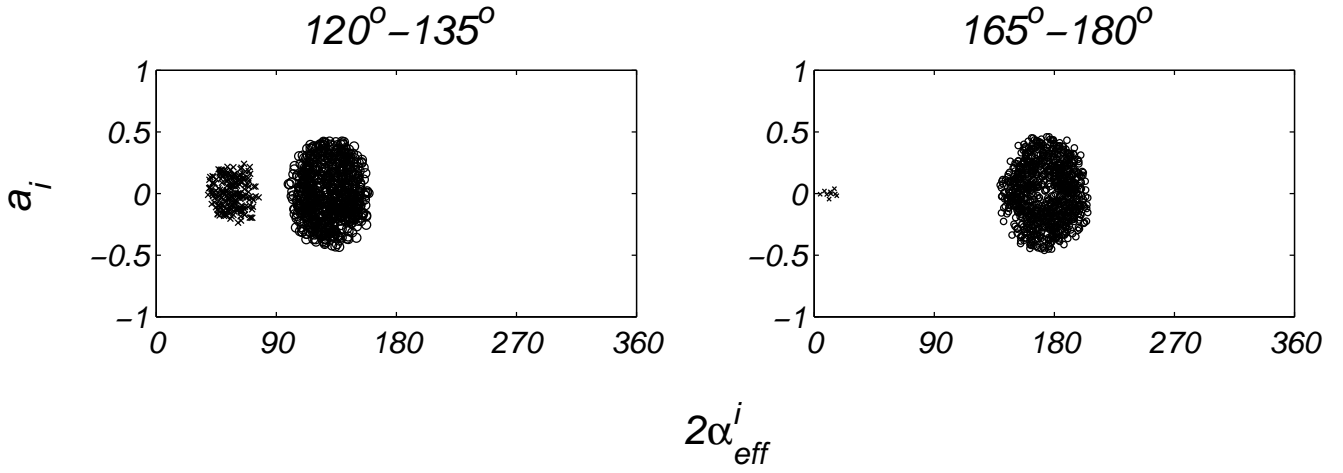


Figure 1: The region in $2\alpha_{eff}^i - a_i$ space ($i = +-, -+$) which is consistent with the theoretical prediction for $|P^i/T^i|$ [Eqs. (12), (14)], for two ranges of 2α , shown above each figure. It is assumed that only $\sin 2\alpha$ is measured, so that 2α is obtained with a twofold ambiguity. (This is the source of the two regions in both figures.) If $\cos 2\alpha$ is also measured, allowing one to obtain 2α without ambiguity, the left-hand region must be removed in both figures.

decays allows one to cleanly extract the CP-violating phase 2α , with no discrete ambiguity. One can also obtain the individual tree (T) and penguin (P) amplitudes in these decays. By comparing the measured value of a particular $|P/T|$ ratio with that predicted by theory, one can detect the presence of physics beyond the standard model. From both a theoretical and experimental point of view, the best $|P/T|$ ratios are those for $B_d^0 \rightarrow \rho^\pm \pi^\mp$. The conservative ranges for these ratios are taken to be $0.05 < |P^i/T^i| < 0.25$ ($i = +-, -+$). The region in $(2\alpha_{eff}^i, a_i)$ parameter space ($2\alpha_{eff}^i$ and a_i are, respectively, the measured indirect and direct CP asymmetries) which corresponds to this range of $|P^i/T^i|$ is relatively small. This therefore provides a good way of detecting the presence of new physics.

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